V & C Patel English School

Half Yearly Exam

Max. Marks: 100

Std.: XII | Date: 15/09/2017

Subject: Mathematics | Time: 3hrs.

General Instructions:-

- *Questions of **Section A** consists of **1 mark** each.*
- *Questions of **Section B** consists of **4 marks** each.*
- *Questions of **Section C** consists of **6 marks** each.*
- *All questions are compulsory.*
- *Use of calculator is not allowed.*

SECTION A

1) If f(x) = $27x^3$ and g(x) = $x^{\frac{1}{3}}$ find gof(x).

2) Write the value of $\tan^{-1}[2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$

3) If matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A^2 = kA then write the value of k.

4) Write the value of the determinant $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

5) Find the points of discontinuity for the function f(x) = $[x]$, -3<x<3

6) Find the slope of the tangent to the curve y = $3x^4$ - 4x at x=4.

SECTION B

7) Find equation of tangents to the curve y = cos(x+y) , $-2\pi \le x \le 2\pi$, that are parallel to the line x + 2y = 0.

8) Find equation of normal to the curve x^2 = 4y which passes through the point (1,2).

9) Prove that f(x) = $\begin{cases} \frac{x}{|x|+2x^2} & ,x \neq 0 \\ k & ,x = 0 \end{cases}$ is discontinuous at x = 0 regardless of the value of k.

10) Find $\frac{dy}{dx}$ for y = $(cosx)^x + (sinx)^{\frac{1}{x}}$.

11) If $x^{16}y^9 = (x^2 + y)^{17}$ then prove that $\frac{dy}{dx} = \frac{2y}{x}$.

12) Using matrices solve the following system of linear equations:
x – y + 2z = 7; 3x + 4y - 5z = -5 ; 2x – y + 3z = 12

13) The management committee of a residential colony decided to award some of its members for Honesty, some for Helping others and some others for Supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for Co-operation and Supervision added to two times the number of awardees for Honesty is 33. If the sum of the number of awardeees for Honesty and Supervision is twice the number of awardees for Helping others, using matrix method find the number of awardees of each category. Apart from these values, namely, Honesty, Co-operation and Supervision suggest one more value which the management of the colony must include for awards.

14) Using elementary transformation find the inverse of the matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

15) Show that $2\tan^{-1}\{tan\frac{\alpha}{2}\tan(\frac{\pi}{4}-\frac{\beta}{2})\} = \tan^{-1}\{\frac{sin\alpha cos\beta}{cos\alpha+sin\beta}\}$

16) If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$ then prove that $9x^2$ - $12xycos\theta + 4y^2 = 36sin^2\theta$.

17) Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f : N \rightarrow Y$ as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

18) If R_1 and R_2 are two equivalence relations in given set A, show that $R_1 \cap R_2$ is also an equivalence relation.

19) Find the intervals in which the function given by $f(x) = sinx + cosx$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

SECTION C

20) Show that $f : N \rightarrow N$ given by $f(x) = \begin{cases} x+1, & x \text{ is odd} \\ x-1, & x \text{ is even} \end{cases}$ is bijective.

21) a) Solve for x, $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

b) Prove $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

22) a) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ verify A^2 - 4A – 5I = 0

b) For $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ verify $(AB)' = B'A'$

23) Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ verify BA = 6I. Use the result to solve

x – y = 3 ; 2x + 3y + 4z = 17 ; y + 2z = 7.

24) Find $\frac{dy}{dx}$ for $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

25) Verify Rolle's theorem for $f(x) = \log(x^2 + 2) - \log 3$ in [-1,1].

OR

Find two positive numbers whose sum is 15 and sum of whose squares is minimum.

26) If length of three sides of a trapezium other than base are equal to 10 cm , then find area of trapezium when it is maximum

*************** ***Good Luck*** ***************