Std.: XI

Subject: Mathematics

Max. Marks: 100
Date: 14/03/2018
Time: 3hrs.

General Instructions:-

- *Questions of Section A consists of 1 mark each.*
- *Questions of Section B consists of 2 marks each.*
- *Questions of Section C consists of 4 marks each.*
- *Questions of Section D consists of 6 marks each.*
- *All questions are compulsory.*
- *Use of calculator is not allowed.*

Section A

1) Find the total number of relations from A to B where $A = \{1,2\}$ and $B = \{a,b\}$.
2) Find the value of $\sin 75°$.
3) Find the latus rectum of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.
4) Translate the following statement into symbolic form " Ram and Shyam went to Delhi".

Section B

5) In a school there are 20 teachers who teach Mathematics or Physics. Of these 12 teach Mathematics and 4 teach both Physics and Mathematics. Find how many teach Physics?
6) let $A = \{1,2\}$, $B = \{1,2,3,4\}$, $C = \{5,6\}$ and $D = \{5,6,7,8\}$. Verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
7) Find the degree measure of the angle subtended at the centre of a circle of radius 100cm by an arc of length 22cm. (use $\pi = \frac{22}{7}$).
8) Express the following in the form of a+ib. $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$
9) How many 6-digit numbers can be formed from the digits 0,1,3,5,7 and 9 which are divisible by 10 and no digit is repeated?
10) Using the binomial theorem expand $(2x - \frac{1}{x})^5$.
11) Find the co-ordinates of foci and vertices of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$.
12) Find the mean of the first 'n' natural numbers.

Section C

13) Let $U = \{1,2,3,4,5,6\}$, $A = \{2,3\}$, $B = \{3,4,5\}$. Find A', B', $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.
14) Find domain and range of the functions: **a)** $f(x) = \sqrt{(x-1)}$ **b)** $f(x) = |x-1|$
15) Prove the following: $\cos(\frac{3\pi}{2} + x)\cos(2\pi + x)[\cot(\frac{3\pi}{2} - x) + \cot((2\pi + x)] = 1$

16) For all $n \geq 1$ prove that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$

17) Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

18) Show that the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where b_1, $b_2 \neq 0$ are

   a) parallel if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$ b) perpendicular if $a_1a_2 + b_1b_2 = 0$

19) Find the equation of the ellipse with the major axis along the x-axis and passing through the points (4,3) and (-1,4).

20) Find the co-ordinates of the point which divides the linesegment joining the points (1,-2,3) and (3,4,-5) in the ratio 2:3   a) internally b) externally

21) Find the derivative of the function $f(x) = \frac{4x+5sinx}{3x+7cosx}$

22) Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$

   and if $\lim_{x \to 1} f(x) = f(1)$ what are possible values of a and b?

23) Show that the following statement is true by method of contrapositive:

   p: If x is an integer and x^2 is even, then x is also even.

Section D

24) A manufacturer has 600 litres of 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

25) Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements

   a) do the words start with P

   b) do all the vowels always occur together

   c) do the vowels never occur together

   d) do the words begin with I and end in P?

26) The second, third and fourth terms in the binomial expansion $(x+a)^n$ are 240, 720 and 1080 respectively. Find x, a and n.

27) Sum of first p, q and r terms of an A.P are a, b and c respectively. Prove that

   $\frac{a(q-r)}{p} + \frac{b(r-p)}{q} + \frac{c(p-q)}{r} = 0.$

OR

In a $\triangle ABC$ prove that $sin2A + sin2B + sin2C = 4 sinA.sinB.sinC$

28) Calculate the mean, variance and standard deviation for the following distribution.

class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

OR

If $f(x) = sin^2x + cos^2x + sinx + cosx$, show that $f'(\frac{\pi}{4}) = 0$

29) A committee of two persons is selected from two men and two women. What is the probability that the committee will have   a) no man b) one man c) two men.

***************Good Luck***************